

Yielding and Flow of Soft-Jammed Systems in Elongation

X. Zhang,¹ O. Fadoul,¹ E. Lorenceau,² and P. Coussot¹

¹Université Paris-Est, Laboratoire Navier (ENPC-IFSTTAR-CNRS), 2 Allée Kepler, 77420 Champs sur Marne, France

²Université Grenoble Alpes, CNRS, LIPhy, 38000 Grenoble, France

(Received 10 September 2017)

So far, yielding and flow properties of soft-jammed systems have only been studied from simple shear and then extrapolated to other flow situations. In particular, simple flows such as elongations have barely been investigated experimentally or only in a nonconstant, partial volume of material. We show that using smooth tool surfaces makes it possible to obtain a prolonged elongational flow over a large range of aspect ratios in the whole volume of material. The normal force measured for various soft-jammed systems with different microstructures shows that the ratio of the elongation yield stress to the shear yield stress is larger (by a factor of around 1.5) than expected from the standard theory which assumes that the stress tensor is a function of the second invariant of the strain rate tensor. This suggests that the constitutive tensor of the materials cannot be determined solely from macroscopic shear measurements.

DOI:

The concept of jamming to characterize materials has flourished and appeared quite useful. Soft-jammed systems, such as foams, emulsions, concentrated suspensions, and colloids, have a structure in which the elements are trapped in potential wells due to their interactions (varying with the distance) with their neighbors and cannot move out due to thermal agitation alone [1]. It is necessary to apply a stress larger than a critical value, i.e., the yield stress (τ_c), to break the structure and induce a flow of the system, otherwise, they behave as solids [1,2].

This concept and its experimental validation, nevertheless, essentially went through simple shear experiments, i.e., a situation where the structure breaks via a relative gliding of material layers along a planar direction. In that case (simple shear), the behavior of granular materials [3], simple yield stress fluids [4], or more complex systems with a behavior depending on the flow history (aging) [5] have been well characterized. However, in many real flow conditions, such as extrusion, blade coating, squeezing, extension, etc., the flow is more complex as it involves some elongational component. In that case, the material undergoes a shrinkage in one direction while being extended in a perpendicular direction. How the yielding properties under such conditions are related to the yield stress observed in simple shear or, more generally, to the material structure, constitutes an open question.

The description of complex flows requires a 3D expression of the constitutive equation. So far, for soft-jammed systems, it has been considered that the stress tensor (\mathbf{T}) is proportional to the strain rate tensor (\mathbf{D}) as for Newtonian fluids but with a factor of proportionality (i.e., the apparent viscosity) depending nonlinearly on the flow intensity. For sufficiently slow flows, i.e., for which the rate-dependent term of the constitutive equation is negligible, this factor is

$\tau_c/\sqrt{-D_{II}}$ where D_{II} is the second invariant of \mathbf{D} ($D_{II} = -\text{tr}\mathbf{D}^2/2$ for an incompressible material), whatever the flow type [6]. This approach has the advantage of predicting the correct yield stress value for simple shear or more complex shear flows [7] and to be consistent with the usual description of the yielding behavior of some solid materials. It is at the basis of models used for the complete numerical simulations of complex yield stress fluid flows [8] but also of granular flows [9]. For a simple uniaxial elongation, this approach predicts that the yielding and slow flow should occur for a normal stress difference (σ) equal to $\sqrt{3}\tau_c$ [10].

In fact, it is not clear at all that soft-jammed systems should follow such a simple 3D behavior. Indeed, the physical origin of this homogeneous 3D constitutive equation for a simple fluid is that all the elements (i.e., molecules) rapidly explore various positions and, thus, can reach the most appropriate ones under some stress. As soon as some significant structural aspect—such as collective arrangement, deformation, or orientation of the elementary components of the fluid—is introduced in a system, it might behave differently, as suggested in Ref. [11]. For example, for polymers whose orientation and length can vary with the flow characteristics, the elongational viscosity may be several orders of magnitude larger than the shear viscosity, whereas the basic linear 3D expression predicts a ratio of 3 [12]. Thus, we can wonder if this homogeneous behavior is still valid for jammed systems in which the structure plays a major role and/or what the relation is between σ and τ_c . Actually, experimental data on such a flow type are scarce, and the conclusions remain fragile because in these experiments, the (supposedly) elongated region was confined in a specific (small) volume of the sample which continuously evolved during the test [13].

85 Finally, contradictory results were obtained, σ being found
 86 between $\sqrt{3}\tau_c$ and $3\sqrt{3}\tau_c$ [13–17].

87 Here we propose an original approach which makes it
 88 possible to get a prolonged elongational flow over a large
 89 range of aspect ratios in the whole volume of material. This
 90 is obtained by considerably reducing the shear stress along
 91 the walls by using smooth surfaces. Data for σ obtained for
 92 different soft-jammed systems are significantly larger than
 93 expected from the standard theory, which suggests that the
 94 yielding and flow properties of jammed systems are more
 95 complex than assumed so far.

96 An approach standard for polymers [18] to obtain an
 97 elongational flow consists to move away two plates in
 98 contact with a cylindrical layer of a soft-jammed system (of
 99 volume Ω and initial thickness h_0) at a velocity $U = dh/dt$,
 100 where h is the current distance between the plates. As h
 101 increases, it is expected that the sample will approximately
 102 keep a cylindrical shape, while its aspect ratio, i.e., h/R ,
 103 where R is the current radius of the (cylindrical) sample,
 104 increases. Such a situation corresponds to a simple uniaxial
 105 elongation. However, for soft-jammed systems and suffi-
 106 ciently slow flows, when the initial aspect ratio is large (say,
 107 $h_0/R_0 \gg 0.1$), the sample immediately evolves as two
 108 conical parts which eventually separate [19,20]. This effect
 109 results from the intrinsic yielding behavior of the material
 110 and the boundary conditions. Since the line of contact is
 111 pinned, an increase of h induces a reduction of the sample
 112 diameter in the central region. Since the (traction) force (F) is
 113 transmitted vertically through the sample, the (mean) normal
 114 stress is larger for a smaller sample diameter. Then the stress
 115 may be larger than σ at some distance from the plates, while it
 116 remains smaller elsewhere. This effect leads to the stoppage
 117 of a growing material volume along each plate while the flow
 118 goes on concentrating in the central region, which finally
 119 leads to the formation and separation of two (approximately)
 120 conical shapes.

121 To damp this effect, we can start from a much smaller
 122 separating distance than the cylinder radius, i.e.,
 123 $h_0/R_0 \ll 1$, since then the relative stress difference in
 124 different sample layers will be smaller. However, in that
 125 case, the radial fluid velocity towards the center V is much
 126 larger than U , since due to sample volume conservation
 127 ($\Omega = \pi R^2 h$): $V = -dR/dt = (R/2h)U$. This induces a
 128 significant shear flow before the fluid separates in two
 129 parts, and finally, F is essentially due to the radial gradient
 130 of pressure induced by this lubricational flow. For slow
 131 flows, F is now proportional to τ_c and scales with $h^{-2.5}$
 132 [10,21]. In fact, under such conditions, the peripheral
 133 interface is unstable and fingering generally occurs
 134 [20,22]. This leads to a force smaller than predicted by
 135 this theory [20] yet with F still approximately scaling with
 136 $h^{-2.5}$ (see Fig. 1). Nevertheless, when h_0/R_0 is increased, F
 137 increases and, in a regime intermediate between well-
 138 developed fingering and direct separation in two cones,
 139 we finally get a force curve close to the theoretical

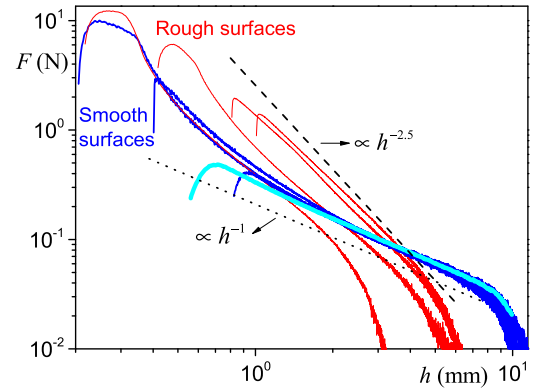


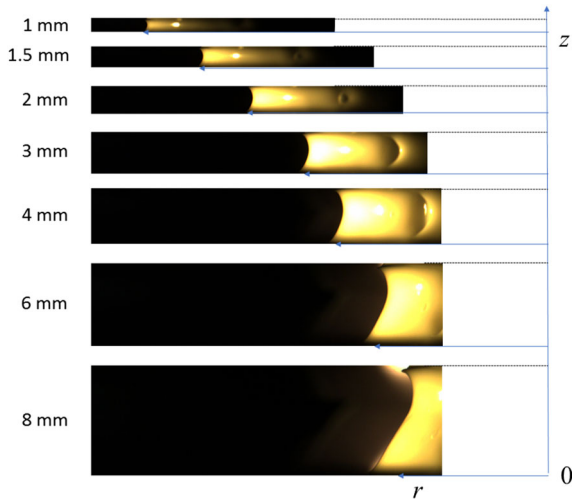
FIG. 1. Force vs distance during a traction experiment for a
 direct emulsion (82%) with rough plates (thin red curves) or
 smooth surfaces (thick dark blue curves) for different initial
 aspect ratios (corresponding to first point of curves on the left)
 at $U = 0.01$ mm/s ($\Omega = 3$ ml). The very thick light blue line
 corresponds to $U/h = \text{const} = 0.01$ s $^{-1}$. The dashed line is
 the lubrication model (see text), and the dotted line is the
 standard theoretical curve for slow elongation.

prediction (see Fig. 1). This anyway does not correspond
 to an elongational flow.

In the above situations, the fluid adherence on the solid
 plate surface is at the origin of the deviation of the flow
 from a simple elongation. In order to suppress or at least
 strongly reduce this adherence, we can use two smooth
 silicon surfaces, along which it is known that due to a wall
 slip effect [23,24], the tangential flow of a soft-jammed
 system is greatly facilitated: the material can move as a
 solid block for a shear stress much smaller than τ_c . Under
 such conditions, we observe that the material keeps a
 cylindrical shape all along the process, with a slight evolution
 of the curvature of the peripheral-free surface (see Fig. 2),
 and this (approximate) cylinder progressively stretches. The
 simplest velocity field compatible with the mass conserva-
 tion and this evolution of the boundaries (neglecting the
 curvature) over more than a decade of h expresses as follows
 in cylindrical coordinates: $v_r = -r\dot{\epsilon}/2$, $v_\theta = 0$, $v_z = z\dot{\epsilon}$,
 with $\dot{\epsilon} = (dh/dt)/h = U/h$ the strain rate. This corresponds
 to a simple uniaxial elongational flow.

Let us now look at the variations of the force $F(h)$
 needed to impose this flow (see Fig. 1). First, it increases
 from a low value, as the material is essentially deformed in
 its solid regime, then it starts to decrease, first rapidly, then
 more slowly and finally follows a decrease as $1/h$ over a
 significant range of h , i.e., 1.5–7 mm. For the analysis
 below, we will not consider the ultimate flow stage at larger
 h , where the force drops to zero (see Ref. [10]).

Interestingly, the force decrease does not depend on h_0
 (see Fig. 1). This differs from flows with rough surfaces for
 which there is an increasing volume of arrested material,
 leading to different force curves for different initial dis-
 tances (see Fig. 1). This suggests that the material defor-
 mation follows the same path in any case (starting from

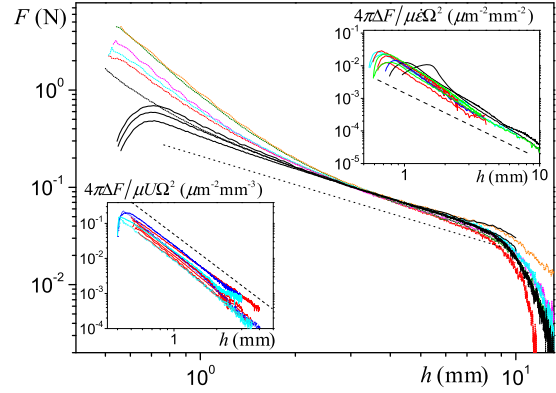


F2:1 FIG. 2. Successive views for different plate distances during
 F2:2 relative motion of two plates in contact via an emulsion (82%)
 F2:3 (yellow region). Here, only one side (relative to the axis) is
 F2:4 (partially) shown (the zones between the yellow regions and the
 F2:5 vertical axis are, in fact, filled with material), the central axis is
 F2:6 shown in blue on the right, and the mean sample radius is
 F2:7 represented by horizontal arrows.

174 different points) and confirms that all the sample volume is
 175 involved in the same flow type at any time.

176 We can also remark that F is initially smaller when U is
 177 decreased and reaches the region $F \propto 1/h$ sooner where all
 178 curves tend to superimpose (see Fig. 3). Thus, we can define
 179 a factor α such that all force curves are situated above α/h
 180 and, for a given h , tend to this value when $U \rightarrow 0$. We also
 181 performed tests by decreasing U when h decreases (i.e.,
 182 $U \propto h$), which allows us to reach this asymptotic curve for
 183 even lower h (see Fig. 3). This has the advantage of
 184 corresponding to a constant strain rate $\dot{\epsilon} = U/h$, which
 185 means that we impose a constant dynamics of elongation to
 186 the material. In that case, we also observe (see Fig. 3) that
 187 the asymptotic curve is reached more rapidly for lower $\dot{\epsilon}$. We
 188 conclude that the minimum force curve that may be reached
 189 for slow flows is $F \propto \alpha/h$, where α is a factor depending on
 190 the sample volume and material characteristics. This situa-
 191 tion is rapidly reached for low $\dot{\epsilon}$ (typically, 0.01 s^{-1}) so that
 192 F follows this law over one decade of h (i.e., here in the range
 193 $0.7\text{--}7 \text{ mm}$) (see Figs. 3 and 4), which corresponds to a range
 194 of aspect ratios varying over one and a half decades [since
 195 $h/R = h/(\Omega/\pi h)^{1/2} \propto h^{1.5}$]. Moreover, it may be shown
 196 that surface tension and gravity effects can be neglected (see
 197 Ref. [10]), which means that the normal force recorded
 198 essentially corresponds to viscous effects in the bulk.

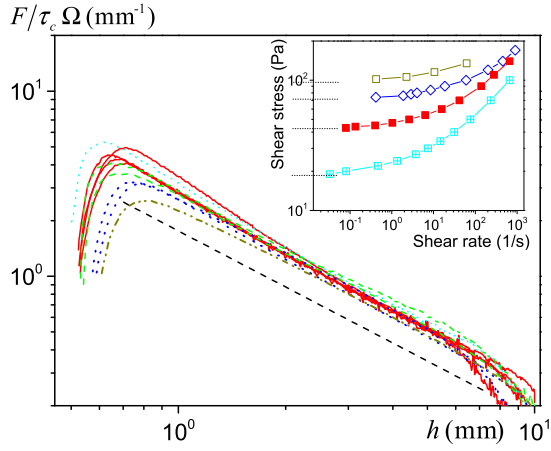
199 The deviation of F from α/h observed at a small h and
 200 more pronounced for large U is somewhat intriguing. It
 201 might be due to the rate-dependent term in the constitutive
 202 equation becoming significant at large $\dot{\epsilon}$. However, we
 203 observe that this effect occurs even under constant $\dot{\epsilon}$ (see
 204 Fig. 3). Moreover, an estimation of the rate-dependent term



F3:1 FIG. 3. Force vs height curve with smooth surfaces during
 F3:2 traction on a yield stress fluid (emulsion 82%) for $\Omega = 3 \text{ ml}$ and
 F3:3 different U [(dotted lines) from bottom to top, $0.01, 0.02, 0.03,$
 F3:4 $0.05, 0.07, 0.1 \text{ mm s}^{-1}$] or $\dot{\epsilon}$ [(continuous curves) from bottom to
 F3:5 top, $0.01, 0.02, 0.03 \text{ s}^{-1}$]. The straight dotted line is the standard
 F3:6 expression for normal force in elongation. The lower inset shows
 F3:7 the rescaled residual force (see text) as a function of the distance
 F3:8 for the above data (red) at constant U and for data obtained with
 F3:9 emulsions at 76% (light blue) and 85% (dark blue) at $U = 0.01$
 F3:10 and 0.05 mm s^{-1} . The dashed line has a slope of -3 in
 F3:11 logarithmic scale. The higher inset shows the rescaled residual
 F3:12 force for data of the main graph (red) at constant $\dot{\epsilon}$ and for
 F3:13 emulsions at 76% (light blue) and 85% (dark blue) at 0.01 and
 F3:14 0.04 s^{-1} , and 82% with a glycerol solution (black) at $\dot{\epsilon} = 0.01$
 F3:15 and 0.04 s^{-1} . Data for an emulsion with smaller droplet size
 F3:16 (i.e., $0.7 \mu\text{m}$) at $\dot{\epsilon} = 0.01$ and 0.04 s^{-1} are also shown (green).
 F3:17 The dotted line has a slope -2 in logarithmic scale. No particular
 F3:18 tendency of variation as a function of U or $\dot{\epsilon}$ may be observed in
 F3:19 the inset graphs.

205 shows that for an elongation, it is always much smaller than
 206 the (constant) plastic term, typically by a factor less than
 207 5% [10]. Such a value is very low in regard to the observed
 208 deviation, which reaches about 300% in some cases (see
 209 Fig. 3). We conclude that in our tests, U should not have a
 210 significant impact on $F(h)$ as long as the flow effectively
 211 corresponds to a uniaxial elongation. Necessarily, the
 212 observed deviation from α/h results from a slightly more
 213 complex flow; for example, in the regions of largest relative
 214 velocity between the material and the solid surfaces, i.e., at
 215 the periphery, some shearing might occur, even if most of
 216 the sample volume still undergoes a pure elongational flow.
 217 These observations, nevertheless, provide information
 218 about the wall slip process in that case.

219 Indeed, let us consider that all occurs as if there were
 220 layers of the interstitial liquid of the material of thickness δ
 221 and viscosity μ situated between the bulk and the solid
 222 surfaces. These liquid layers essentially allow us to strongly
 223 reduce the (shear) adherence of the material to the solid
 224 surface, but they also transmit the normal force needed to
 225 induce the bulk flow. Since $\delta \ll R$, these layers undergo a
 226 (lubricational) shear flow due to the relative motion of their
 227 boundaries: the solid surface on one side and the interface



F4:1 FIG. 4. Rescaled force vs height during traction tests at a
 F4:2 constant strain rate (0.01 s^{-1}) for emulsions at a concentration of
 F4:3 82% for different volumes (1, 2, 3, and 4 ml) (continuous red
 F4:4 curves); emulsions at concentrations 76% (dotted light blue)
 F4:5 (3 ml), 85% (dotted dark blue) (1 and 3 ml), and 87% (brown
 F4:6 dash dotted line) for 3 ml; a Carbopol gel (dashed green) for 1 and
 F4:7 3 ml. The dotted straight line corresponds to $\sigma = \sqrt{3}\tau_c$. Reproducibility
 F4:8 tests show that the uncertainty on these data is 10%
 F4:9 (see Ref. [10]). The inset shows the flow curves in simple shear
 F4:10 for the three emulsions (from bottom to top) 76%, 82%, 85%,
 F4:11 87%. The dotted lines show the level of yield stress as deduced
 F4:12 from creep tests. The uncertainty on these values is less than 5%
 F4:13 (see Ref. [10]).

228 with the shrinking bulk on the other side. The resulting
 229 normal force from the induced pressure gradient writes
 230 $F = -p_R \Omega / h + \mu \Omega^2 U / 4\pi h^3 \delta^2$. The pressure term p_R
 231 (negative, relative to the ambient pressure) *a priori* results
 232 from the Laplace pressure drop associated with the curva-
 233 ture of the liquid-air interface. The validity of this expres-
 234 sion may be checked on our data by withdrawing from each
 235 experimental force curve a $-p_R \Omega / h$ term fitted to the data
 236 at large h values (i.e., when the second term is negligible).
 237 The residual force (i.e., $\Delta F = F + p_R \Omega / h$) effectively
 238 varies as a function of h , Ω , μ , and U as predicted by
 239 the above expression, i.e., $4\pi \Delta F / \mu \Omega^2 U = 1/h^3 \delta^2$ for
 240 constant U and $4\pi \Delta F / \mu \Omega^2 \dot{\epsilon} = 1/h^2 \delta^2$ for constant $\dot{\epsilon}$
 241 (see the insets of Fig. 3). The value for δ may, thus, be
 242 deduced from the comparison of the data with the theo-
 243 retical expression.

244 From this analysis, we surprisingly find a constant value
 245 for the wall slip layer thickness under any conditions in our
 246 range of tests (see the insets of Fig. 3): $\delta = 9 \pm 3 \mu\text{m}$. This
 247 means that the liquid volume available for slip continuously
 248 adjusts during traction, an effect likely due to the reentrance
 249 of the liquid in the material structure as it shrinks. Note that
 250 this reentrance, which could affect the flow characteristics
 251 and, thus, modify the second term of the force expression,
 252 has apparently a negligible impact. Another surprising
 253 result is that δ is several orders of magnitude larger than
 254 in simple shear for the same kinds of material (i.e.,

255 $35 \pm 15 \text{ nm}$; see Ref. [24]) but with no clear relation with
 256 a characteristic length of the material structure (here droplet
 257 size) (see the top inset of Fig. 3). This suggests that wall
 258 slip in an elongational process has a different nature than in
 259 simple shear.

260 Let us come back to the $1/h$ regime for the bulk flow. We
 261 can now compute a normal stress $\sigma = F/\pi R^2$, which, due
 262 to mass conservation, may also be expressed as Fh/Ω . Our
 263 results show that at sufficiently low $\dot{\epsilon}$, σ is a constant equal
 264 to $\sigma = \alpha/\Omega$. Further tests (see Fig. 4) show that α is
 265 proportional to Ω , which means that σ is independent of Ω .
 266 Finally, these tests allow us to measure a quantity, i.e., the
 267 normal stress σ , which is independent of the current aspect
 268 ratio and the size of the material, as long as the sample
 269 remains cylindrical. This is the *normal stress difference*

270 associated with a simple uniaxial elongational flow at
 271 sufficiently low $\dot{\epsilon}$ (see Ref. [10]), which here appears to
 272 be an intrinsic property of the soft-jammed system. Since
 273 this value is the minimal normal stress that must be applied
 274 to impose such an elongational flow, this is the (simple
 275 uniaxial) elongation yield stress.

276 Note that in contrast with usual approaches which studied
 277 elongational flows in a localized volume of the sample
 278 during a transient flow [13–17], here we have *a priori* a
 279 straightforward measure of the normal stress needed to
 280 impose a prolonged elongational flow in the whole sample
 281 volume and over a significant range of aspect ratios.

282 In addition, we independently determined the (shear)
 283 yield stress for our different materials from a well-
 284 controlled series of precise creep tests in shear geometry
 285 which allow us to clearly distinguish the liquid and the
 286 solid regimes and the critical stress (τ_c) associated with the
 287 transition. These data also provide the simple shear flow
 288 curve (see the inset of Fig. 4), whose validity was checked
 289 through tests with other procedures and equipment (see
 290 Ref. [24]). Further traction tests then show that for a given
 291 material type, σ is simply proportional to τ_c (see Fig. 4).
 292 The factor of proportionality is equal to $1.5\sqrt{3}$ for emulsions
 293 and Carbopol gels at various concentrations (see Fig. 4),
 294 which is 1.5 larger than predicted by the standard theory.
 295 This factor is even larger for two more complex materials
 296 (mustard and ketchup), i.e., of the order of $2.5\sqrt{3}$ (see
 297 Ref. [10]). These results show that the assumption of a factor
 298 depending only on the second invariant in the constitutive
 299 equation, and, thus, being equal to $\sqrt{3}$, is not valid. A
 300 possibility is that the parameters of this constitutive equation
 301 depend on the third invariant of \mathbf{D} [i.e., $D_{III} = \det(\mathbf{D})$], as
 302 suggested in Ref. [15], e.g., with now the extrastress tensor
 303 expressing as $\tau_c \mathbf{D} / [(-D_{II})^{1/2} + \alpha D_{III}^{1/3}]$ in slow flows.
 304 For the emulsions and Carbopol gels, $\alpha = -0.46$ allows us to
 305 well represent the data. One may also think of using other
 306 plasticity criteria for expressing this first term of the con-
 307 stitutive equation.

308 This shows that the standard simple view of jamming
 309 described with a homogeneous approach (i.e., second

310 invariant of the stress tensor) is not valid. Our results
 311 suggest that the 3D expression of their constitutive equation
 312 is more complex than suggested so far and cannot leave
 313 apart the specificities of the material structure, and more
 314 particularly, the physical origin of jamming, e.g., squeezed
 315 objects or particles interacting at a distance. This also
 316 implies that appropriate models of the constitutive equa-
 317 tion, in particular, for yielding and slow flow regime, have
 318 to be developed.

319 **3** This work has benefited from a French government grant
 320 managed by ANR within the frame of the national program
 321 Investments for the Future ANR-11-LABX-022-01.

324
 325 [1] A. J. Liu and S. R. Nagel, *Nature (London)* **396**, 21 (1998).
 326 [2] P. Coussot, *Rheometry of Pastes, Suspensions and Granular*
 327 *Materials* (Wiley, New York, 2005).
 328 [3] G. D. R. Midi, *Eur. Phys. J. E* **14**, 341 (2004).
 329 [4] G. Ovarlez, S. Cohen-Addad, K. Krishan, J. Goyon, and P.
 330 Coussot, *J. Non-Newtonian Fluid Mech.* **193**, 68 (2013).
 331 [5] G. Ovarlez, S. Rodts, X. Chateau, and P. Coussot, *Rheol.*
 332 *Acta* **48**, 831 (2009).
 333 **4** [6] J. G. Oldroyd, *Proc. Cambridge Philos. Soc.* **43**, 100 (1947);
 334 J. M. Piau, *Tech. Ing. A* **710**, A711 (1979).
 335 [7] G. Ovarlez, Q. Barral, and P. Coussot, *Nat. Mater.* **9**, 115
 336 (2010).
 337 [8] P. Saramito and A. Wachs, *Rheol. Acta* **56**, 211 (2017).
 338 [9] P. Jop, Y. Forterre, and O. Pouliquen, *Nature (London)* **441**,
 339 727 (2006).
 340 [10] See the Supplemental Material at [http://link.aps.org/](http://link.aps.org/supplemental/10.1103/PhysRevLett.000.000000)
 341 [supplemental/10.1103/PhysRevLett.000.000000](http://link.aps.org/supplemental/10.1103/PhysRevLett.000.000000), which
 342 includes information about the surface characteristics,

rheometry, basic theory, procedure for the elongation tests, 343
 reproducibility, additional data, lubricational flow, flow in 344
 the slip layer, surface tension effects, gravity effects, and the 345
 relative importance of the different stress terms. 346
 [11] J. M. Brader, M. E. Cates, and M. Fuchs, *Phys. Rev. Lett.* 347
101, 138301 (2008). 348
 [12] C. W. Macosko, *Rheology Principles, Measurements and* 349
Application (VCH, New York, 1994). 350
 [13] J. Boujlel and P. Coussot, *Soft Matter* **9**, 5898 (2013). 351
 [14] G. German and V. Bertola, *Phys. Fluids* **22**, 033101 (2010). 352
 [15] K. Niedzwiedz, H. Buggisch, and N. Willenbacher, *Rheol.* 353
Acta **49**, 1103 (2010); L. Martinie, H. Buggisch, and N. 354
 Willenbacher, *J. Rheol.* **57**, 627 (2013). 355
 [16] N. Louvet, D. Bonn, and H. Kellay, *Phys. Rev. Lett.* **113**, 356
 218302 (2014). 357
 [17] M. K. Tiwari, A. V. Bazilevsky, A. L. Yarin, and C. M. 358
 Megaridis, *Rheol. Acta* **48**, 597 (2009). 359
 [18] G. H. McKinley and T. Sridhar, *Annu. Rev. Fluid Mech.* **34**, 360
 375 (2002). 361
 [19] P. Coussot and F. Gaulard, *Phys. Rev. E* **72**, 031409 (2005); 362
 N. J. Balmforth, N. Dubash, and A. C. Slim, *J. Non-* 363
Newtonian Fluid Mech. **165**, 1147 (2010). 364
 [20] Q. Barral, G. Ovarlez, X. Chateau, J. Boujlel, B. Rabideau, 365
 and P. Coussot, *Soft Matter* **6**, 1343 (2010). 366
 [21] G. H. Covey and B. R. Stanmore, *J. Non-Newtonian Fluid* 367
Mech. **8**, 249 (1981); G. H. Meeten, *Rheol. Acta* **41**, 557 368
 (2002). 369
 [22] D. Derks, A. Lindner, C. Creton, and D. Bonn, *J. Appl.* 370
Phys. **93**, 1557 (2003). 371
 [23] M. Cloitre and R. T. Bonnecaze, *Rheol. Acta* **56**, 283 372
 (2017). 373
 [24] X. Zhang, E. Lorenceau, P. Basset, T. Bourouina, F. Rouyer, **5** 374
 J. Goyon, and P. Coussot, *Phys. Rev. Lett.* **119**, 208004 375
 (2017). 376
 377